From statistical modeling to AI-integrated inverse Gaussian process: a comprehensive review for prognostics and health management

Liangliang Zhuang[a], Yizhong Ma[a], Jianjun Wang[a], Rong Pan[b], Ancha Xu[c,d,*]

[a] *School of Economics and Management, Department of Management Science and Engineering, Nanjing University of Science and Technology, Nanjing, 210094, China*
[b] *School of Computing and Augmented Intelligence, Arizona State University, Tempe, AZ85281 USA*
[c] *School of Statistics and Data Science, Zhejiang Gongshang University, Hangzhou 310018, China*
[d] *Collaborative Innovation Center of Statistical Data Engineering, Technology & Application Zhejiang Gongshang University, Hangzhou 310018, China*

Abstract

In prognostics and health management (PHM), degradation modeling plays a central role in reliability analysis and lifetime prediction. The inverse Gaussian (IG) process has recently attracted increasing attention for its ability to describe monotonic and cumulative degradation with heavy-tailed behavior, analytical tractability, and clear physical interpretability. Meanwhile, the rapid development of artificial intelligence (AI) has created new opportunities to combine statistical modeling with learning-based approaches in reliability analysis. This paper presents a comprehensive review of IG-process-based degradation modeling, covering its theoretical foundations, model extensions, parameter estimation, and diagnostic methods. Applications in accelerated degradation test design, burn-in test, remaining useful life prediction, and maintenance optimization are systematically summarized. Recent progress on AI-integrated IG frameworks is also reviewed and critically assessed. In addition, key challenges and research opportunities are discussed to guide future developments in intelligent PHM.

Keywords: Artificial intelligence, Degradation modeling, Inverse Gaussian process, Maintenance decision, Remaining useful life prediction

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*Corresponding author

Email addresses: zhuangll@njust.edu.cn (Liangliang Zhuang), yzma-2004@163.com (Yizhong Ma), jjwang@njust.edu.cn (Jianjun Wang), rong.pan@asu.edu (Rong Pan), xuancha@mail.zjgsu.edu.cn (Ancha Xu)

1. Introduction

1.1. Background

Reliability is a fundamental aspect of life-cycle management, ensuring the safe, efficient, and sustainable operation of engineering assets. With the advent of *Industry 4.0*, manufacturing systems have become increasingly digital and automated, resulting in higher complexity and stronger data dependence. Consequently, the coexistence of heterogeneous data sources and time-varying operating conditions has posed new challenges for reliability analysis. Meanwhile, advances in sensing and monitoring technologies now enable continuous recording of degradation signals during operation. Compared with lifetime observations, degradation data capture richer temporal patterns that support early fault detection, condition assessment, and insight into failure mechanisms (He et al., 2025b; Wang and Tang, 2025; Zhang et al., 2025a). As a result, degradation-based reliability analysis has become an essential component of modern operating and maintenance systems (Meeker et al., 2021; Lawless, 2011; Ouyang et al., 2026).

In this context, prognostics and health management (PHM) has emerged as a key paradigm for intelligent maintenance. PHM employs degradation models to predict the remaining useful life (RUL) of systems and supports condition-based maintenance (CBM), forming a closed-loop framework that links data acquisition, prognostics, and decision-making (Xu and Wang, 2025). From an experimental perspective, accelerated degradation tests (ADTs) further enhance reliability assessment by using rational stress design and observation planning to obtain early information on lifetime behavior. Consequently, stochastic degradation process models play a central role in PHM, bridge data monitoring, health assessment, and maintenance optimization, while providing a unified probabilistic foundation for lifetime modeling, reliability evaluation, and maintenance decision-making.

1.2. Impact of Artificial Intelligence on PHM

The rapid development of artificial intelligence (AI) has profoundly transformed the paradigm of PHM. With the advent of the industrial Internet, intelligent sensing, and edge computing, large-scale, heterogeneous, and high-frequency monitoring data can now be continuously collected from complex engineering systems, providing a rich information base for health assessment and lifetime prediction. AI techniques—such as deep neural networks, recurrent and graph neural networks, as well as federated and transfer learning—enable end-to-end modeling from raw signals to fault diagnosis, health assessment, and RUL prediction. They have been successfully applied in various engineering domains (Jia et al., 2024; Wang et al., 2024; Shen et al., 2025a,b). For instance, Nguyen et al. (2022) developed a Bayesian neural network that predicts RUL with quantified confidence intervals, and Guo et al. (2023) proposed a federated learning framework for tool wear prediction using convolutional autoencoders to enhance global model training. Such studies demonstrate the potential of AI to capture nonlinear degradation behaviors and complex dependencies, pushing PHM toward system-level, data-driven intelligence.

Nevertheless, purely AI-based frameworks still face critical challenges in reliability-oriented applications. Most deep models lack physical and statistical interpretability and cannot provide credible uncertainty quantification, which limits their trustworthiness in risk-sensitive maintenance decision-making. In addition, degradation data available from industrial systems are often limited in size, noise, or incomplete, leading to restricted generalization and robustness of AI-driven models. Addressing these issues requires modeling frameworks that preserve the learning capacity of AI while maintaining probabilistic consistency and statistical rigor—an emerging and important research direction for intelligent PHM (He et al., 2025a,c).

1.3. Literature Review

Recent studies on degradation modeling are typically categorized into two main types: physics-based models and data-driven models, as illustrated in Figure 1. Physics-based models rely on material fatigue, corrosion, or electrochemical mechanisms to characterize the evolution of degradation. They offer strong interpretability but limited adaptability across diverse operating conditions. In contrast, data-driven models, including statistical models and AI–based models, have become prevalent due to their flexibility and scalability across applications.

As discussed in Section 1.2, AI-based methods excel at learning complex and nonlinear degradation behaviors, but their data dependence and limited interpretability constrain their use in reliability-oriented PHM. By comparison, statistical modeling approaches play a central role in reliability analysis because they have clear theoretical foundations and interpretable models. Representative models include general path models and stochastic process models. The former incorporate covariates, random effects, and unit-to-unit variability into their structure, enabling flexible representation of unit-specific degradation behaviors, as

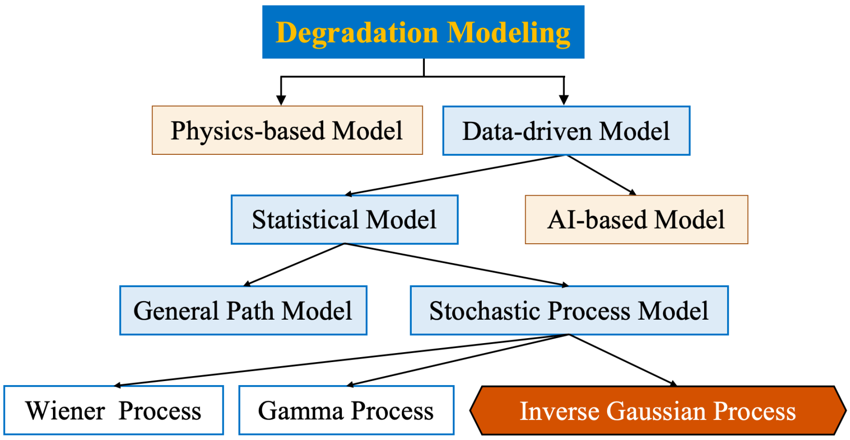
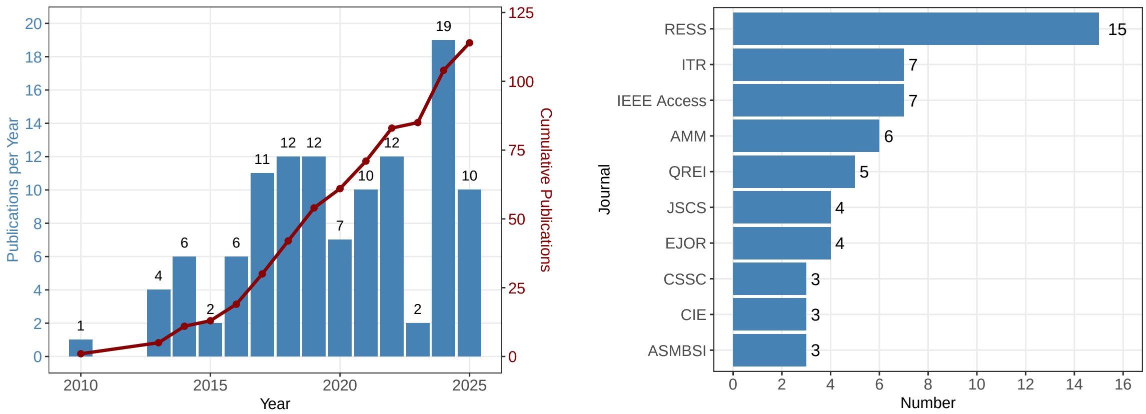


Figure 1: Overview of degradation modeling approaches in reliability and PHM.

exemplified by the hierarchical degradation model in Lu et al. (2021) and the dynamic covariate model in Hong et al. (2015). Among stochastic process models, the Wiener, gamma, and inverse Gaussian (IG) processes are the most widely used. The Wiener process is capable of capturing non-monotonic degradation with random fluctuations (Zhang et al., 2018), whereas the gamma process, inherently monotonic, has been extensively applied to corrosion, wear, and insulation aging (Tung and Tseng, 2019; Zhou et al., 2024; Song, 2024; Yuan et al., 2021). However, its light-tailed property limits its capacity to represent highly dispersed or heavy-tailed degradation behavior. The IG process, a non-decreasing Lévy process, preserves monotonicity while modeling heavy-tailed increments through the inverse Gaussian distribution. With its closed-form first-passage time (FPT) distribution, it provides an effective and interpretable tool to model monotonic degradation and reliability behavior (Peng et al., 2014).

Figure 2 provides a bibliometric overview of research on the IG process. We first queried the Web of Science database using the keyword "inverse Gaussian process", and then expanded the search through cross-citation analysis. After screening, a total of 114 related publications were obtained as of September 2025. The earliest systematic application of the IG process in degradation modeling appeared in Wang and Xu (2010), after which its use expanded steadily across various engineering domains. As shown in Figure 2(a), research on the IG process has grown steadily overall, with notable fluctuations in annual publications and a temporary peak observed in 2024. Figure 2(b) highlights that most studies were published in leading reliability journals such as Reliability Engineering & System Safety and IEEE Transactions on Reliability. This concentration indicates that IG-process research



(a) Annual publication trend

(b) Main publishing journals

Figure 2: Bibliometric analysis of IG process studies: (a) annual publication trend; (b) main publishing journals.

is primarily driven by the reliability/PHM community and remains closely aligned with reliability-oriented modeling and decision-making problems.

1.4. Framework and Contributions

Although numerous studies have applied the IG process to degradation modeling, most existing work remains case-specific and lacks a unified methodological perspective. To the best of our knowledge, this is the first comprehensive review dedicated to the IG process in PHM. The purpose of this paper is to establish a systematic methodological framework for IG-based degradation modeling, summarize recent developments, and identify open challenges and future research trends. Figure 3 presents the overall structure of this paper. We progressively move from theoretical foundations to statistical inference and then to engineering applications; meanwhile, motivated by the rapid development of AI, recent studies have begun to integrate learning-based techniques with IG modeling to enhance modeling flexibility while preserving its probabilistic structure. Specifically, this paper:

(i) revisits the theoretical foundations of the IG process and its probabilistic properties;

(ii) reviews methodological extensions of the IG process to account for random effects, measurement errors, covariates, and multiple performance characteristics (PCs) degradation, and summarizes statistical inference methods for parameter estimation and model evaluation;

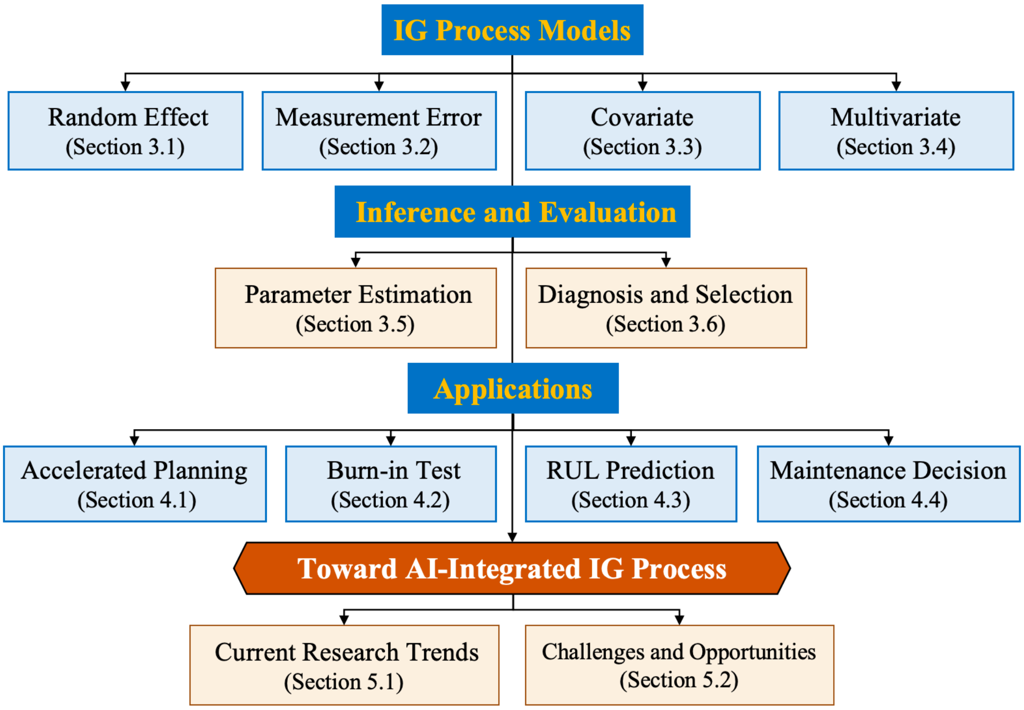


Figure 3: Overall structure of this paper, illustrating the relationships among IG-process modeling, inference, applications, and its evolution toward AI-integrated PHM.

(iii) summarizes engineering applications of the IG process by focusing on four representative tasks: ADT design, burn-in test, RUL prediction, and maintenance optimization;

(iv) discusses emerging AI-integrated IG frameworks, highlighting recent progress, challenges, and future research opportunities.

The remainder of this paper is organized as follows. Section 2 outlines the basic formulation of the IG process. Section 3 reviews its model extensions and inference methods, while Section 4 presents representative engineering applications. Section 5 explores AI-integrated IG processes and future research directions, while Section 6 concludes the paper.

2. Theoretical Foundations and Modeling Framework

Let $\{Y(t), t > 0\}$ denote a degradation process characterized by the IG process. It satisfies the following properties: (i) $Y(0) = 0$ with probability one. (ii) For $t > s > u > 0$, the increments $Y(t) - Y(s)$ and $Y(s) - Y(u)$ are independent. (iii) Each increment follows an IG distribution, that is,

$$Y(t) - Y(s) \sim \text{IG}\left(\mu\Delta\Lambda, \lambda\left(\Delta\Lambda\right)^2\right), \tag{1}$$

where μ is the drift parameter, λ is the volatility parameter, and $\Delta\Lambda = \Lambda(t) - \Lambda(s)$, $\Lambda(t)$ is a non-decreasing function of t with $\Lambda(0) = 0$. Accordingly, the probability density function of IG $(\mu\Lambda(t), \lambda\Lambda(t)^2)$ is given by

$$f_{\text{IG}}(y \mid \mu, \lambda) = \left(\frac{\lambda\Lambda(t)^2}{2\pi y^3}\right)^{1/2} \exp\left[-\frac{\lambda}{2y}(y/\mu - \Lambda(t))^2\right], \quad y > 0. \tag{2}$$

The mean and variance of the IG process are

$$\text{E}[Y(t)] = \mu\Lambda(t), \qquad \text{Var}[Y(t)] = \frac{\mu^3\Lambda(t)}{\lambda}.$$

This formulation provides clear physical interpretations of the IG process: the parameter μ represents the degradation rate, λ characterizes the path variability, and $\Lambda(t)$ acts as a shape function describing the cumulative effect of time. When $\Lambda(t) = t$, the process exhibits stationary, independent increments; a nonlinear $\Lambda(t)$ yields a non-stationary process capable of capturing time-dependent degradation increments behavior. Typical choices of $\Lambda(t)$ include the power-law form $\Lambda(t; \alpha) = t^\alpha$, and the exponential form $\Lambda(t; \alpha) = \exp(\alpha t) - 1$, where α is parameter to be estimated (Zhai et al., 2025; Meeker et al., 2021).

Based on the above definition, the IG process can be further used in reliability analysis. In particular, the FPT represents the time when the degradation path first reaches the failure threshold $\mathcal{D}$, defined by

$$T_{\mathcal{D}} = \inf\{t \geq 0 : Y(t) \geq \mathcal{D}\}.$$

Since $\Pr(T_{\mathcal{D}} > t) = \Pr(Y(t) < \mathcal{D})$, the reliability function can be directly derived from the cumulative distribution of $Y(t)$. Thus, the distribution and density of $T_{\mathcal{D}}$ are given by

$$\begin{aligned}
F_{T_{\mathcal{D}}}(t) &= \Phi\left(\sqrt{\frac{\lambda}{\mathcal{D}}}\left(\Lambda(t) - \mathcal{D}/\mu\right)\right) - e^{2\lambda\Lambda(t)/\mu}\Phi\left(-\sqrt{\frac{\lambda}{\mathcal{D}}}(\Lambda(t) + \mathcal{D}/\mu)\right), \\
f_{T_{\mathcal{D}}}(t) &= \sqrt{\frac{\lambda}{\mathcal{D}}}\,\phi\left(\sqrt{\frac{\lambda}{\mathcal{D}}}(\Lambda(t) - \mathcal{D}/\mu)\right)\Lambda'(t) - \frac{2\lambda}{\mu}\Lambda'(t)e^{2\lambda\Lambda(t)/\mu}\Phi\left(-\sqrt{\frac{\lambda}{\mathcal{D}}}(\Lambda(t) + \mathcal{D}/\mu)\right) \\
&\quad + \sqrt{\frac{\lambda}{\mathcal{D}}}\,\Lambda'(t)e^{2\lambda\Lambda(t)/\mu}\,\phi\left(-\sqrt{\frac{\lambda}{\mathcal{D}}}(\Lambda(t) + \mathcal{D}/\mu)\right),
\end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density and cumulative distribution functions of the standard normal distribution, and $\Lambda'(\cdot)$ is the derivative of $\Lambda(\cdot)$.

In engineering applications, a normal approximation is commonly used when $\lambda\Lambda(t)$ is sufficiently large (Ye et al., 2014). In this case, $Y(t)$ can be approximated as $Y(t) \approx \mathrm{N}(\mu\Lambda(t), \mu^3\Lambda(t)/\lambda)$, and the distribution of $T_{\mathcal{D}}$ can be approximated by

$$F_{T_{\mathcal{D}}}(t) \approx 1 - \Phi\left(\frac{\mathcal{D} - \mu\Lambda(t)}{\sqrt{\mu^3\Lambda(t)/\lambda}}\right) = \Phi\left(\sqrt{\frac{\lambda}{\mu}}\sqrt{\Lambda(t)} - \frac{\mathcal{D}\sqrt{\lambda/\mu^3}}{\sqrt{\Lambda(t)}}\right),$$

which corresponds to a Birnbaum–Saunders (BS) distribution (Ye and Chen, 2014; Tang and Chang, 1995). The p-th quantile of $T_{\mathcal{D}}$ can then be expressed as

$$q_p = \Lambda^{-1}\left(\frac{\mu}{4\lambda}(z_p + \sqrt{z_p^2 + 4\mathcal{D}\lambda/\mu^2})^2\right),$$

where z_p is the p-th quantile of the standard normal distribution, and $\Lambda^{-1}(\cdot)$ denotes the inverse of the shape function.

With the IG process defined above, we briefly contrast it with the Wiener and gamma processes. The Wiener process is driven by Brownian motion, allows nonmonotonic sample paths, and is therefore suitable for degradation processes with reversible fluctuations. By contrast, the gamma and IG processes are monotone-increasing by construction and are more appropriate for modeling degradation in the form of wear and cumulative damage. From a structural perspective, both the gamma and IG processes can be viewed as continuous limits of compound Poisson processes, but their different shock-size distributions lead to distinct tail behaviors. In particular, the IG process can exhibit a heavier right tail than the gamma process, enabling better accommodation of observed variability and occasional larger degradation increments. In addition, Ye and Chen (2014) showed that, compared with the gamma process, the IG process is more flexible in incorporating covariates and random effects, which can be attributed to its inverse relationship with the Wiener process.

3. Model Extensions and Inference Methods

The IG process has been extended to consider random effects (Section 3.1), measurement errors (Section 3.2), covariate effects (Section 3.3), and multivariate PCs (Section 3.4). Methods for parameter estimation (Section 3.5) and model evaluation (Section 3.6) have also been reviewed.

3.1. IG Process with Random Effects

Random effects have been incorporated into the IG process to account for unit-level heterogeneity. Depending on how the random effects are introduced, three representative models have been developed.

3.1.1. Random-Drift Model

The random-drift (RD) model is suited for cases where the degradation rate varies across units, while the variability level remains constant. Let the drift parameter μ be treated as a random effect, with a common volatility parameter λ shared among all units:

$$Y(t) \sim \mathrm{IG}\left(\mu\,\Lambda(t), \lambda\,\Lambda(t)^2\right), \qquad \mu^{-1} \sim G(\boldsymbol{\theta}),$$

where $G(\boldsymbol{\theta})$ is a distribution ensuring $\mu > 0$, and $\boldsymbol{\theta}$ is the corresponding parameter vector. A common choice is a truncated normal prior on μ^{-1} (Ye and Chen, 2014; Peng et al., 2014), so that the conditional posterior remains truncated normal and preserves the Markov property (Ye and Chen, 2014). This property implies that given observations Y, the update of $\mu^{-1} \mid Y$ depends only on the latest measurement, enabling efficient online prediction. The RD model has been applied to a variety of datasets—such as hydraulic piston pumps (Sun et al., 2021), light-emitting diodes (Chen et al., 2019), and GaAs lasers (Shen et al., 2019a)—and further extended to maintenance optimization problems (Chen et al., 2015).

Beyond the truncated normal assumption, some studies adopt a normal distribution for computational simplicity, allowing straightforward integration with the expectation–maximization (EM) algorithm or recursive estimation methods (Fang et al., 2022; Xu et al., 2020, 2025). This approach has been applied to RUL prediction for laser devices, cabin door locks, and cutting tools (Pan et al., 2016; Yan et al., 2025; Huang et al., 2021). However, this assumption is appropriate only when $\mu \gg \sigma^2$, ensuring that the distribution approximates a truncated normal and remains positive. Time-varying random effects have also been explored by specifying $\mu^{-1} \sim \mathrm{N}(\varrho, t/\sigma_\mu^2)$, where the variance grows linearly with time to capture dynamic uncertainty in the drift parameter (Li et al., 2022; Wu et al., 2020). This formulation, derived from a Brownian-driven process, reflects early-stage variability and long-term stabilization, offering a realistic representation of degradation in mission-oriented systems.

Another line of work introduces mixture distributions to capture unit-to-unit variability. For example, Xu et al. (2020) assumed the random-effect parameter follows a mixture of normals, $\mu^{-1} \sim \sum_{k=1}^{K} p_k \mathrm{N}\left(\nu_k, \sigma_k^2/\lambda\right)$, where p_k are weights, and ν_k, σ_k^2 represent the mean and variance of subgroup k. This formulation captures latent subpopulations and offers greater flexibility when a single distribution cannot adequately represent unit-to-unit variability. Finally, skew-normal distributions have been adopted to capture asymmetric unit-level variability (Hao et al., 2019; Chen et al., 2020a), typically defined as $\mu^{-1} \sim \mathrm{SN}(\xi, \omega^2, \vartheta)$, where ξ, ω^2, and ϑ denote the location, scale, and skewness parameters, respectively. When $\vartheta = 0$, the model reduces to a normal distribution, while varying ϑ flexibly controls skewness. Table 1 summarizes representative random-effect distributions used in the RD model, together with their parameter specifications and relevant studies.

Table 1: Representative random-effect distributions used in the RD model.

Distribution	Specification	References
Truncated normal	$\mu^{-1} \sim \mathrm{TN}(\omega, \kappa^{-2})$	Sun et al. (2021); Duan et al. (2018); Chen et al. (2015); Ye and Chen (2014); Peng et al. (2014); Liu et al. (2014); Feng et al. (2025); Chen et al. (2019); Shen et al. (2019a)
Normal (time-invariant)	$\mu^{-1} \sim \mathrm{N}(a, b)$	Zhuang et al. (2024); Xu et al. (2025); Fang et al. (2022); Xu et al. (2020); Wu et al. (2020); Pan et al. (2016); Yan et al. (2025); Huang et al. (2021); Wang et al. (2020)
Normal (time-varying)	$\mu^{-1} \sim \mathrm{N}(\varrho, t/\sigma_\mu^2)$	Li et al. (2022); Wu et al. (2020)
Mixture of normals	$\mu^{-1} \sim \sum_{k=1}^{K} p_k \mathrm{N}(\nu_k, \sigma_k^2/\lambda)$	Xu et al. (2020)
Skew-normal	$\mu^{-1} \sim \mathrm{SN}(\xi, \omega^2, \vartheta)$	Hao et al. (2019); Chen et al. (2020a); Liang et al. (2024)

3.1.2. Random-Volatility Model

The random-volatility (RV) model applies when units exhibit comparable average degradation rates but differ in noise levels or stochastic variability. The degradation process is defined as

$$Y(t) \sim \mathrm{IG}\left(\mu \Lambda(t), \lambda \Lambda(t)^2\right), \qquad \lambda \sim G(\boldsymbol{\theta}),$$

where heterogeneity is captured through the volatility parameter λ while the drift μ remains constant across units. A common assumption is $\lambda \sim \mathrm{Gamma}(\delta, \gamma)$, where δ and γ denote the shape and scale parameters, respectively. More importantly, the gamma distribution is conjugate to the IG process, such that the conditional posterior $\lambda \mid Y$ also follows a gamma distribution. This conjugacy enables explicit posterior updating and greatly simplifies parameter estimation and computation (Wang and Xu, 2010; Teng and Wang, 2018). It should be noted that, unlike the RD model, the RV model no longer preserves the Markov property—its predictive distribution depends on the entire observation sequence. Nevertheless, the gamma conjugate structure maintains good computational tractability and engineering applicability. For instance, Zheng et al. (2024); Peng et al. (2019); Wang and Xu (2010) employed the RV model for ADTs and verified its effectiveness under varying stress conditions; Zhang et al. (2024b); Liang et al. (2026) applied it to RUL prediction, demonstrating improved predictive accuracy. Ye et al. (2014); Wang et al. (2017) incorporated the RV structure into experimental design to highlight its advantages during test planning.

3.1.3. Random Drift–Volatility Model

The random drift–volatility (RDV) model simultaneously accounts for the unit-to-unit heterogeneity in both the drift and volatility parameters. Two major formulations have been proposed. The first, introduced by Ye and Chen (2014), defines the degradation process as

$$Y(t) \sim \mathrm{IG}\left(\mu\Lambda(t), \lambda\mu^2\Lambda(t)^2\right), \qquad \mu \sim \mathrm{TN}(\omega, \kappa^{-2}).$$

This model describes cases where faster degradation is accompanied by higher volatility. Although the conditional distribution of μ given the observations is analytically complex, its moments can be computed recursively, which facilitates efficient implementation of the EM algorithm for parameter estimation. Peng et al. (2014) later developed a Bayesian analysis of the RDV model and demonstrated its performance using GaAs laser degradation data. The second, proposed by Peng (2015), introduces a hierarchical structure to jointly characterize the dependence between the drift and volatility parameters:

$$Y(t) \sim \mathrm{IG}\left(\mu\Lambda(t), \lambda\Lambda(t)^2\right), \qquad \mu^{-1} \mid \lambda \sim \mathrm{N}(\omega, \sigma^2/\lambda), \qquad \lambda \sim \mathrm{Gamma}(\tau, \beta).$$

This model exploits the normal–gamma conjugacy for efficient inference and results in a Student-t marginal. The model also generalizes the RD and RV structures as special cases: when $\sigma^2 \to 0$ and $\mu = 1/\omega$, it corresponds to the RV model; when $\tau \to \infty$ and λ is fixed, it corresponds to the RD model. Using the hierarchical structure proposed by Peng (2015), Teng and Wang (2018) applied the model to accelerated degradation data for reliability assessment, while Peng et al. (2022) employed it in optimal test design to improve the adaptability of planning to unit-to-unit variability. Fan et al. (2024) developed a fully Bayesian hierarchical model, implementing Markov chain Monte Carlo (MCMC)-based inference and achieving RUL prediction.

3.1.4. Other Random-Effect Models

Beyond the classical structures, several alternative models have been proposed to capture unit-to-unit variability. One stream follows the idea of *frailty*, introducing latent random factors into the IG process to account for unobserved variation. For example, Morita et al. (2021a) introduced a multiplicative frailty term into the IG process by allowing the latent frailty variable to act on the increment-level hazard representation, with the frailty following either a gamma or an IG distribution. Zhang et al. (2024a) proposed an IG model incorporating a frailty term that follows a generalized IG distribution to quantify unobservable heterogeneity, and validated the approach using fatigue crack degradation data. In another application, Cha et al. (2021) modeled mixed frailty factors into the drift parameter to distinguish weak and strong components, facilitating warranty policy optimization. Similarly, Morita (2017) proposed a latent class model that assumes the population consists of subgroups (e.g., weak and normal units), each governed by different IG parameters, thereby revealing the structural heterogeneity within the population and enhancing lifetime prediction precision.

Overall, these approaches demonstrate the flexibility of the IG process in capturing latent heterogeneity and subgroup differences, thereby enabling its application to multi-source data analysis and system-level reliability modeling.

3.2. IG Process with Measurement Errors

Degradation measurements are often affected by limited instrument precision, environmental disturbances, or manual inspection uncertainty, sensor noise. In the presence of such measurement errors, the observed degradation data can be expressed as

$$Y_i(t_{ij}) = X_i(t_{ij}) + \epsilon_{ij}, \qquad X_i(t) \sim \mathrm{IG}\big(\mu\Lambda(t_{ij}),\, \lambda\Lambda(t_{ij})^2\big)\,, \quad \epsilon_{ij} \sim G(\boldsymbol{\phi}),$$

where $X_i(t_{ij})$ denotes the true degradation path of unit i at observation time t_{ij}, and ϵ_{ij} represents the measurement error, following a distribution $G(\boldsymbol{\phi})$ that can be specified according to the application.

Most studies assume normally distributed errors, $\epsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$, owing to simplicity and tractability (Hao et al., 2019; Chen et al., 2020b). For example, Rodríguez-Picón et al. (2019) proposed a likelihood-based deconvolution method that models the convolution of the degradation and error distributions, correcting estimation bias and improving lifetime prediction for fatigue crack data. Extensions have introduced more realistic error structures. Sun et al. (2021) introduced a state-dependent measurement error model, in which the error variance is related to the actual degradation state, allowing the variance to vary with the degradation level. This formulation captures the fact that measurements tend to become less stable as wear progresses. Another line of research modeled correlated measurement errors using multivariate normal structures, where the covariance matrix captures dependencies among repeated measurements under different batches, stress levels, or time points (Qin et al., 2013; Ma et al., 2020; Cui et al., 2024). More recently, Chen et al. (2025) relaxed the normality assumption by allowing multivariate skew-normal distribution to handle heavy-tailed or asymmetric noise in bearing degradation data.

Overall, incorporating measurement errors into the IG framework enhances the realism of degradation modeling and improves the accuracy of parameter estimation. Existing studies have considered a range of error structures, including Gaussian, state-dependent, correlated, and non-Gaussian forms, though with increased model complexity. Future research may focus on developing more efficient estimation methods and integrating multi-source information under noisy conditions.

3.3. IG Process with Covariates

Degradation processes are often influenced by stress, environmental, and operational factors. Accordingly, IG models with covariates have been developed to incorporate these external variables into the model parameters. Existing studies can be broadly classified into three categories.

(1) Stress-Accelerated Covariates. In ADT, degradation data are obtained under elevated stress conditions and then extrapolated to normal use to estimate product lifetime. Modeling the relationship between stress and degradation rate is therefore essential for reliable lifetime prediction. Within the IG framework, a common approach is to express the drift parameter μ or the volatility parameter λ as a function of stress, such as the exponential form $\mu = \exp(a + bx)$ (Wang and Xu, 2010; Ye and Chen, 2014), or dual-acceleration structures where both μ and λ depend on stress (Jiang et al., 2024; He et al., 2021; Tang and Zhou, 2025). Alternatively, stress effects can be embedded in the shape function $\Lambda(t)$ through cumulative exposure or proportional degradation rate models (Duan and Wang, 2018b; Wang et al., 2016).

(2) Environmental and Mission Covariates. Under complex operating conditions, degradation can also be affected by environmental and mission-related factors. For instance, Li et al. (2022) incorporated mission time and maintenance state into the drift term of the IG process within a task-oriented CBM framework, achieving coordinated optimization between task planning and maintenance decisions. Das et al. (2024) introduced machining parameters such as cutting speed, feed rate, and depth of cut into the degradation rate function in an exponential form, and employed Bayesian MCMC estimation to analyze tool wear data under various operating conditions. Lou et al. (2022) further developed a bidirectional wear model that integrates load and medium factors into the IG process to capture wear evolution under complex frictional environments.

(3) Mechanism- and State-Dependent Covariates. Another research direction seeks to integrate physical mechanisms or state variables into the IG process to enhance interpretability and flexibility. Peng et al. (2019) proposed a transformed IG model that introduces covariates through an age- or state-dependent monotonic transformation, enabling nonstationary degradation to be analyzed within the IG framework. Chen et al. (2022b) employed high-dimensional image features as covariates to characterize material degradation, achieving improved modeling accuracy.

Overall, covariate-driven IG models can be broadly classified into three categories: stress-accelerated, environmental and mission-based, and mechanism- or state-dependent. These models extend the IG framework from traditional stress testing to more realistic operational and physical conditions, enhancing its applicability to complex degradation scenarios in PHM.

3.4. Multivariate IG Processes

Many engineering systems show joint degradation in multiple interdependent PCs (Yi et al., 2025), such as polymer coatings (Lu et al., 2021) and heavy machine tools (Peng et al., 2016b), among others. To capture such dependency, researchers have developed multivariate IG process models, which can be broadly classified into three categories.

(1) Multivariate Stochastic-Process Methods. These methods characterize the statistical dependence among multiple PCs through explicit covariance modeling or correlated random-effect structures. For example, Qu et al. (2024) developed a bivariate IG process model to describe fatigue crack growth data, using the covariance structure to capture correlation between degradation paths. Meanwhile, Fang et al. (2022) introduced correlated random effects, assuming μ^{-1} follow a multivariate normal distribution, thereby jointly modeling unit heterogeneity and the dependence among multiple PCs. Such methods are straightforward but mostly rely on multivariate normal assumptions, limiting their flexibility in capturing complex dependencies. As dimensionality increases, computational burden also grows substantially (Yin et al., 2026).

(2) Copula-Based Methods. In this framework, each marginal distribution of a PC follows an IG process, while their dependency structure is modeled separately using a copula function. Suppose the system has d PCs, and denote by $F_j(y_j)$ the IG marginal distribution of the jth PC; then

$$P(Y_1 \leq y_1, \ldots, Y_d \leq y_d) = C(F_1(y_1), \ldots, F_d(y_d); \theta_c)\,,$$

where $C(\cdot)$ is a copula function and θ_c is the dependency parameter. Several representative studies have applied the copula-based IG framework in different contexts. For example, Duan et al. (2018) incorporated a Franck copula structure into a bivariate IG framework to describe the dependence between two PCs under a Bayesian setting; Fu et al. (2024) further adopted a Vine-copula formulation to capture complex dependencies in electronic systems. These models have also been applied to other engineering systems, such as hydraulic seals (Chen et al., 2022a) and metallic components (Rodríguez-Picón et al., 2019). Overall, the copula-based framework offers strong flexibility in modeling nonlinear and asymmetric dependencies but remains computationally demanding in high dimensions and sensitive to the choice of copula family.

(3) Latent Shared-Factor Methods. These methods introduce shared latent factors across multiple IG processes to capture the interdependence among multiple PCs. For example, Zhou et al. (2017) proposed an additive stochastic-process framework that decomposes multiple IG processes into shared and individual components to model environmental dependence among correlated corrosion defects in pipelines. Building on this concept, Zhuang et al. (2025) developed a multivariate reparameterized IG model with explicit common terms to achieve an additive and scalable structure. Feng et al. (2025) proposed a bivariate IG model, in which a latent factor jointly influences two PCs for reliability analysis of the wet clutch system. Compared with copula-based methods, shared-factor approaches offer greater physical interpretability: when PCs are affected by common environmental or mechanistic influences, they can more robustly capture dependencies in small-sample settings, though forcing shared factors under unrelated mechanisms may lead to model bias.

Beyond these main approaches, some studies have coupled IG processes with other failure mechanisms, such as competing-risk models (Jin et al., 2020) and load-sharing systems (Liu et al., 2016), offering a system-level framework for complex degradation interactions. Overall, the key challenge in multivariate IG modeling lies in effectively modeling the dependency structure among multiple PCs: stochastic-process-based methods are intuitive but computationally intensive in higher dimensions; copula models are flexible but less interpretable; and shared-factor approaches offer a balance between simplicity and interpretability, but may introduce bias when the assumed common factors do not reflect the true dependency mechanisms. Representative studies of these approaches are listed in Table 2. To facilitate comparison, Table 3 summarizes representative IG-process extension models discussed in this section, highlighting their key assumptions, strengths, limitations, and typical application scenarios.

Table 2: Representative studies on multivariate IG process modeling.

Model	Idea	References
Multivariate Stochastic-Process	Directly extend IG to multivariate forms	Qu et al. (2024); Fang et al. (2022)
Copula-Based	Characterize dependence using copula functions	Duan and Wang (2018a); Rodríguez-Picón et al. (2017); Duan et al. (2018); Peng et al. (2016b,a); Liu et al. (2014); Chen et al. (2022a); Fu et al. (2024); Fuqiang et al. (2021); Rodríguez-Picón et al. (2019); Zhou et al. (2017)
Latent Shared-Factor	Introduce latent factors to represent shared environmental effects	Zhou et al. (2017); Zhuang et al. (2025); Feng et al. (2025)

3.5. Parameter Estimation Methods

Parameter estimation forms the foundation of IG-based degradation modeling and reliability analysis, and existing methods can be grouped into three categories.

(1) Maximum Likelihood Estimation. Maximum likelihood (ML) method is the most widely used approach for parameter estimation in IG models. It estimates model

Table 3: Comparison of representative IG-process extension models.

Models	Key assumptions	Strengths	Limitations	Typical applications
Random effect	Unit-level heterogeneity represented by latent random effects	Capture population heterogeneity and support individual-level inference	Sensitive to random-effect distributional assumptions; identifiability issues under limited data	Heterogeneous populations; accelerated degradation testing
Measurement error	Observed degradation subject to additive measurement noise	Separate measurement noise from degradation dynamics, improving parameter estimation	Identifiability issues under sparse measurements; sensitive to error-distribution assumptions	Noisy sensing environments; condition monitoring
Covariate	Effects of covariates on degradation are specified through predefined functional forms	Quantify operating-condition effects and support extrapolation across conditions	Require informative covariates; sensitive to the assumed covariate-effect form	Accelerated degradation testing; reliability prediction under varying conditions
Multivariate	Correlated PCs modeled with a specified dependence structure	Capture cross-PC dependence and improve diagnostic and prognostic performance	Computational efficiency degrades rapidly as dimensionality increases	Complex systems with multi-dimensional PCs

parameters by maximizing the likelihood function given the observed data. Numerous studies have demonstrated the efficiency and consistency of ML estimation (Portela et al., 2025; Liang et al., 2026). The ML method is computationally efficient and analytically tractable but may deteriorate in small samples or in the presence of latent variables. In such cases, the EM algorithm provides a practical alternative, iteratively updating parameter estimates until convergence. When closed-form solutions are available, the algorithm converges rapidly (Fang et al., 2022; Xu et al., 2020); otherwise, numerical approximations are adopted (Zhuang et al., 2024; Sun et al., 2021). Recent studies have also introduced improved initialization strategies for enhanced stability (Zhuang et al., 2025). Overall, the EM algorithm offers a flexible inference framework for IG models with latent structures, though its convergence and optimality are sensitive to initialization (Ye and Chen, 2014).

(2) Bayesian Inference. Bayesian methods introduce prior distributions and combine them with observed data to form posterior estimates, offering a coherent framework for quantifying parameter uncertainty (Zhu et al., 2026). They are particularly effective for small samples, heterogeneous systems, and models with hierarchical or latent structures. Depending on prior choices and computational strategies, existing Bayesian implementations of IG models fall into three main types: (i) those adopting conjugate or empirical priors for analytical tractability (Fan et al., 2024; Li et al., 2017a), (ii) those employing objective priors (e.g., Jeffreys or reference) to reduce subjectivity and improve robustness in small samples (Guan et al., 2019; He et al., 2018), and (iii) simulation-based inference relying on MCMC techniques for complex hierarchical structures (Peng et al., 2014; Zhuang et al., 2024). Overall, Bayesian inference provides a unified probabilistic framework for complex IG models, naturally integrating prior knowledge with data to quantify joint uncertainty in parameters and lifetimes. Compared with ML or EM methods, it offers greater robustness and interpretability, but at the expense of higher computational cost and sensitivity to convergence.

(3) Other Estimation Methods. Beyond ML and Bayesian approaches, several alternative estimation methods have been proposed. For small-sample inference, generalized pivotal quantity methods construct distribution-free statistics to achieve exact and robust interval estimation (Zheng et al., 2024; Jiang et al., 2024, 2022; Chen and Ye, 2018). Bootstrap resampling approximates the sampling distribution of estimators through repeated resampling, providing practical uncertainty quantification for reliability models with latent or heterogeneous structures (Zhuang et al., 2025; Xu et al., 2020; Peng, 2015; Wang and Xu, 2010). Such approaches enhance modeling flexibility while maintaining interpretability, though their theoretical properties require further investigation.

3.6. Model Diagnosis and Selection

After parameter estimation, model diagnosis and selection are essential to verify the adequacy and applicability of the IG-based degradation model. Common diagnostic tools include goodness-of-fit (GOF) tests, residual analysis, and information criteria for model comparison. Ye and Chen (2014) proposed two statistical GOF methods for IG processes. First, a χ^2-based test for degradation increments: if the increment $y_{ij} = Y_i(t_{ij}) - Y_i(t_{i,j-1})$ follows $\mathrm{IG}(\Lambda_{ij}, \eta\Lambda_{ij}^2)$, then $\zeta_{ij} = \eta(y_{ij} - \Lambda_{ij})^2/y_{ij} \sim \chi_1^2$. When the parameters are replaced by

ML estimates, the statistic $\hat{\zeta}_{ij}$ should approximately follow a chi-square distribution with one degree of freedom (χ_1^2), allowing visual assessment through a χ^2 Q–Q plot. Similarly, Q–Q plots can be adapted for random-effect models through appropriate modifications. Second, a GOF test can be conducted based on the probability integral transform. Given a failure threshold $\mathcal{D}$, pseudo failure times $\hat{T}$ are first obtained through nonlinear regression. The transformed variables $u = F_{T_{\mathcal{D}}}(\hat{T})$ should then follow a uniform distribution $U(0, 1)$ if the model is correctly specified. This uniform transformation can be visually assessed using a Q–Q plot to evaluate model adequacy. Similar GOF tests have been applied in multivariate and mixed-effect IG models (Fang et al., 2022; Zhuang et al., 2024; Xu et al., 2020).

Model selection in IG-based degradation models commonly relies on information criteria and predictive performance. The Akaike information criterion (AIC) is commonly used to compare alternative model formulations: $\text{AIC} = 2\dim(\Theta) - 2\ell(\hat{\Theta})$, where $\dim(\Theta)$ denotes the number of parameters and $\ell(\hat{\Theta})$ the maximized log-likelihood. Although AIC has been widely applied, for example in evaluating random-effect or measurement-error structures (Hao et al., 2019; Fang et al., 2022), it may be biased in small-sample settings. For this reason, the corrected AIC (AICc) or the Bayesian information criterion is often recommended as a more reliable alternative (Chen et al., 2022a). In addition to information-criterion based selection, likelihood ratio tests (LRT) are commonly employed for hypothesis testing in nested IG models (Zhuang et al., 2025). In the Bayesian framework, posterior predictive checks have been employed to evaluate model adequacy (Fan et al., 2024), and cross-validation is often used to assess predictive performance (Ye and Chen, 2014; Pang et al., 2024). Overall, model diagnosis and selection seek to balance GOF, model complexity, and predictive capability, ensuring that IG-based degradation models are both statistically rigorous and practically applicable.

4. Applications of IG Processes in PHM

With the modeling and inference developments reviewed above, IG processes have been applied broadly in PHM across diverse engineering systems. These application studies are typically built on degradation observations collected from different systems (e.g., lithium-ion batteries and metallic materials). To provide a concise data-oriented overview, Table 4 summarizes representative datasets and related references. This section reviews representative IG-process applications by typical PHM tasks, including ADT design (Sections 4.1) and burn-in testing (Section 4.2), RUL prediction (Section 4.3), and maintenance decision-making (Section 4.4).

Table 4: Common degradation datasets and representative references in IG-process-based PHM applications.

Dataset	Representative references
Stress relaxation	Zhou et al. (2025); Jiang et al. (2024); Ma et al. (2021); Tang and Zhou (2025); Pang et al. (2024); Mosayebi Omshi and Shemehsavar (2019)
Fatigue crack growth	Zhuang et al. (2025); Morita et al. (2021a); Xu et al. (2020); Zhang et al. (2024a)
Lithium-ion battery capacity fade	Zhuang et al. (2024); Xu et al. (2025); Peng et al. (2019); Liang et al. (2024)
GaAs laser aging	Fan et al. (2024); Morita et al. (2021a); Yang et al. (2024); Peng et al. (2022); Portela et al. (2025); Morita et al. (2021b); Wei et al. (2024)
Output power of heavy machine tools	Peng et al. (2017); Duan et al. (2018); Peng et al. (2016b,a); Duan et al. (2023)
Integrated circuit power decrease	Zheng et al. (2024); Xu et al. (2025); Jiang et al. (2022); Xu et al. (2020)
Milling tool wear	Zhang et al. (2024b); Huang et al. (2021); Das et al. (2024)
Return-oil flow of hydraulic piston pumps	Sun et al. (2021); Ma et al. (2019)
Carbon-film resistor resistance drift	He et al. (2019, 2021)
Bearing vibration signals	Chen et al. (2025)
Coating material aging	Fang et al. (2022)

4.1. Accelerated Degradation Test Design

For highly reliable products, lifetime testing under normal stress conditions is often extremely time-consuming and costly. ADT aims to shorten test duration by applying higher stress levels while maintaining the same failure mechanism as in normal conditions. Building on the general form in Eq. (1), the IG process parameters μ and λ are modeled as stress-dependent functions $\mu(x)$ and $\lambda(x)$, typically following exponential or power-law relationships (Limon et al., 2017). Common optimality criteria include D-, A-, V-, c-, and

M-optimal designs, targeting different objectives such as parameter estimation efficiency or prediction variance minimization. In the Bayesian framework, these can be extended to maximize expected information gain or minimize predictive risk. Within the IG-process framework, existing ADT studies can be grouped into three categories according to how stress levels are modeled and applied.

(1) Constant-Stress ADT. CSADT is the most widely adopted form of ADT owing to its simplicity, stable failure mechanism, and ease of data analysis. Within the IG framework, Ye et al. (2014) proposed an optimal CSADT design minimizing the asymptotic variance of lifetime quantiles under given stress and sample constraints. Wang et al. (2017) derived an M-optimal design criterion to improve parameter estimation precision, while Wu et al. (2019) combined D- and V-optimal objectives to resolve conflicts among single-criterion designs. Mosayebi Omshi and Shemehsavar (2019) further optimized CSADT plans based on D-optimality, and Zhou et al. (2025) focused on measurement-time optimization, demonstrating substantial efficiency gains in experimental design.

(2) Step-Stress ADT. Step-stress ADT (SSADT) increases the stress level in a stepwise manner during the experiment to further accelerate degradation while keeping the same failure mechanism. Two modeling frameworks are commonly used for IG-based SSADT: a) the proportional degradation rate model, where the shape function $\Lambda(t)$ is shared across stress levels while the drift and volatility parameters (μ, λ) scale proportionally with stress, enabling unified modeling across conditions; and b) the cumulative exposure model, which treats total degradation as the accumulation of stress-dependent effects, providing a physically interpretable formulation (Wang et al., 2016; Duan and Wang, 2018b). Recent studies have extended SSADT objectives beyond lifetime estimation to multi-objective and Bayesian frameworks. For example, Li et al. (2017b, 2018) introduced Bayesian optimal designs based on Kullback–Leibler divergence, quadratic loss, and Bayesian D-optimality; Tang and Zhou (2025) proposed an M-optimal design maximizing the minimum predictive variance for robust performance; and Mosayebi Omshi and Azizi (2022) considered the tampered-drift effect under D-optimality. Further advances include bivariate correlated degradation modeling (Qu et al., 2024), and sequential Bayesian designs (Li et al., 2017a).

(3) Hybrid and Extended ADT. Under the IG process, several extensions of conventional ADT frameworks have been developed to integrate complementary reliability information. Ma et al. (2021) proposed a hybrid ALT–ADT design that jointly models short-term degradation and long-term lifetime behavior, using a V-optimality criterion to minimize the variance of lifetime quantiles while optimizing stress levels, inspection times, and sample allocation. Beyond ADT–ALT integration, Wang et al. (2018) developed an accelerated-stress reliability acceptance test combining degradation modeling with qualification testing through acceleration-factor analysis, providing an efficient alternative to conventional acceptance testing. These extended IG-based ADT frameworks enhance reliability assessment across multiple time scales but increase model and design complexity.

4.2. Burn-in Test

Burn-in test is a common reliability screening strategy for high-reliability products. It subjects units to accelerated operation before service to identify and remove early-failure items, thereby reducing the in-service failure rate. The key design challenge is to determine the optimal burn-in duration t_b and decision rule that balance the removal of defective units against the associated cost and lifetime reduction.

A common modeling approach uses a mixture IG process to describe heterogeneous populations consisting of "early-failure" and "normal" units. Conditional on the latent group indicator Z, the degradation process follows

$$Y(t) \mid Z = z \sim \mathrm{IG}\big(\mu_z \Lambda(t),\, \lambda_z \Lambda(t)^2\big)\,, \quad Z \sim \mathrm{Bernoulli}(p),$$

where $Z = 1$ indicates an early-failure unit, $Z = 0$ a normal unit, and p the proportion of early failures. The survival probability at time t_b is

$$S(t_b) = (1 - p)\, P(T_0 > t_b) + p\, P(T_1 > t_b),$$

which provides the basis for deriving failure probabilities, cost functions, and expected lifetime to determine the optimal burn-in duration.

Under the IG framework, research on burn-in testing has mainly focused on several aspects. Early studies adopted mixture IG models to distinguish early-failing units from normal units, and to determine the optimal burn-in duration that minimizes total cost. For instance, Zhang et al. (2014) proposed an optimal burn-in policy under a mixed population, while Morita (2017) further developed decision rules for unit classification and introduced an economic cost model to optimize burn-in duration and cutoff thresholds. Morita et al. (2021b) extended this framework by incorporating copula functions into the mixture IG model to capture dependencies among multiple PCs, showing that correlation structure significantly affects burn-in decisions. In recent years, Bayesian frameworks have been adopted to improve inference under small samples and high uncertainty. Wei et al. (2024) proposed a joint two-dimensional burn-in and warranty strategy integrating degradation and failure data, where the posterior distribution enables dynamic and adaptive decision-making.

Research on IG-based burn-in modeling has evolved from simple mixture formulations to multivariate dependence structures and Bayesian decision frameworks. Owing to its capability to characterize early degradation initiation and its clear probabilistic interpretation, the IG process provides a useful basis for optimizing burn-in strategies. However, most existing studies remain theoretical or simulation-based. Future work could focus on integrating large-scale monitoring data and intelligent optimization methods to support real-time burn-in decisions and validate the approach in practical engineering systems.

4.3. RUL Prediction

The IG process provides a probabilistic framework for predicting the RUL of degrading systems, given a failure threshold $\mathcal{D}$, the RUL at time t_0 is defined as

$$\text{RUL}(t_0) = \inf\{u > 0 : Y(t_0 + u) - Y(t_0) \geq \mathcal{D} - Y(t_0) \mid Y(t_0)\}.$$

Owing to the IG increment property, the RUL distribution can be derived analytically or approximated by a BS distribution under the basic framework. Beyond the basic framework, extensions involving random effects, measurement errors, covariates, and multivariate structures have led to diverse treatments of RUL inference. We summarize these in four categories.

(1) RUL Prediction with Random Effects. With the introduction of random effects, RUL inference no longer relies on conditional distributions with fixed parameters. Instead, it requires integration over the distribution of unit-specific parameters to obtain the marginal lifetime distribution. Most existing studies achieve this by introducing suitably specified random effects, such as normal, truncated-normal, or skew-normal priors, thereby retaining analytical tractability and yielding explicit RUL distributions for efficient reliability prediction (Sun et al., 2021; Hao et al., 2019; Xu et al., 2020). In contrast, more complex stochastic structures, such as two-stage degradation processes (Zhuang et al., 2024) or IG models with latent frailty factors (Morita et al., 2021a), lack closed-form lifetime distributions and therefore rely on numerical integration for RUL prediction. While such models can more realistically describe multi-phase or system-level dependencies, they substantially increase computational cost and hinder real-time implementation.

(2) RUL Prediction with Measurement Error. When measurement noise is present, RUL inference cannot directly rely on the observed degradation trajectory but must be conditioned on the latent true degradation state. Early studies corrected measurement errors at the observation level and then performed RUL prediction using the reconstructed degradation path (Rodríguez-Picón et al., 2019). Later studies explicitly embedded measurement errors into the degradation model, allowing joint inference of the true state and its remaining lifetime distribution (Sun et al., 2021).

(3) RUL Prediction with Covariates. When degradation depends on covariates, RUL prediction is performed conditionally on these variables, often by linking model parameters to stress or usage conditions for lifetime extrapolation. Ye et al. (2014) developed a framework that links IG process parameters to stress levels, which has since been extended with alternative stress–parameter mappings for lifetime extrapolation (He et al., 2021, 2018; Peng, 2015). Beyond stress covariates, Chen et al. (2022b) introduced microscopic image features as covariates to build a high-dimensional IG model capturing material-level microstructural effects. Because the resulting lifetime distribution is analytically intractable, RUL prediction was approximated numerically.

(4) RUL Prediction with Multiple PCs. In multi-PC degradation scenarios, system lifetime is defined as the minimum of the FPTs of all PCs, and RUL prediction is thus performed within a joint multivariate IG framework accounting for both marginal behaviors and dependencies. Copula-based approaches retain IG marginals and capture inter-PC dependencies via copula functions (Duan et al., 2018; Chen et al., 2022a; Rodríguez-Picón et al., 2019), while latent-factor or correlated random-effect models describe dependence through shared latent variables (Fang et al., 2022; Feng et al., 2025). In all cases, the joint RUL distribution requires high-dimensional integration, which is generally evaluated through numerical or simulation-based approximation.

4.4. Maintenance Decision-Making

In CBM or predictive maintenance (PdM) frameworks, optimizing maintenance strategies is essential for improving system availability and minimizing life-cycle costs. The IG process provides a physically interpretable degradation model whose analytically tractable first-passage distribution allows direct integration into maintenance decision models. Under a standard cost-based framework, the expected maintenance cost at time t is defined as

$$C(t) = c_p \cdot P(T > t) + c_f \cdot P(T \leq t),$$

where c_p and c_f represent the costs of preventive and corrective maintenance, respectively, and T denotes the failure time. Within this setting, the conditional distribution of the IG process enables explicit derivation of optimal inspection intervals or replacement thresholds that minimize long-term average cost.

Based on this framework, several studies have explored its applications in CBM optimization. Chen et al. (2015) incorporated an IG random-effects process into CBM to derive an optimal maintenance policy with a monotone control-limit structure. Peng et al. (2016b) developed a bivariate copula–IG model to jointly infer incomplete degradation information and predict future states for maintenance decision-making in multi-PC systems. Wu et al. (2020) proposed a dynamic-threshold multi-objective optimization framework to balance premature failure risk and maintenance cost, while Li et al. (2022) embedded a task-oriented preventive replacement threshold within an IG-based PdM model. Zhuang et al. (2024) proposed an adaptive replacement strategy within a two-phase IG framework, where model parameters and RUL estimates are dynamically updated to support real-time maintenance decision-making. To account for maintenance history effects, Huynh (2021) incorporated adaptive PdM into the IG framework, where random effects capture the influence of previous maintenance on subsequent degradation. Based on a semi-regenerative process, the study showed that the proposed strategy effectively mitigates early failures and reduces maintenance costs. Portela et al. (2025) further developed a non-constant imperfect maintenance model by embedding maintenance efficiency into the IG degradation path, optimizing strategies under time-dependent repair effectiveness.

The IG process has also been applied to the joint optimization of maintenance and warranty strategies. Cha et al. (2021) proposed a mixed IG model for heterogeneous populations to design renewable warranty policies, balancing inspection frequency, maintenance thresholds, and warranty cost. Shang et al. (2018) developed an integrated warranty–maintenance framework based on the IG process to maximize manufacturer profit and determine optimal post-warranty PdM strategies under competitive conditions. At the system level, IG-based models have been extended to multi-component and mechanism-driven maintenance strategies. Zhu and Hao (2021) developed a unified optimization model for multi-component systems that integrates component replacement and reordering policies to balance cost and system availability. From a mechanistic perspective, Lou et al. (2022) proposed a bidirectional wear model based on the IG process to analyze the impact of wear parameters on degradation prediction and replacement thresholds, offering guidance for corrosion- and erosion-related maintenance.

Overall, IG-based maintenance and warranty decision research has evolved from unit-level threshold optimization to frameworks that account for maintenance history, warranty coordination, and system-level dependencies. Its analytical tractability provides a consistent probabilistic basis for modeling and optimizing various maintenance policies.

5. Toward AI-Integrated IG Process for PHM

Figure 4 summarizes the methodological evolution of IG-process-based degradation modeling. Building on classical IG models and their subsequent statistical extensions, recent years—driven by increasing system complexity and richer data availability—have witnessed a growing interest in integrating AI techniques with the IG process. Since around 2023, such AI-integrated approaches have been explored to enhance modeling flexibility and data adaptivity while retaining the probabilistic structure of the IG process, thereby better supporting prediction accuracy and uncertainty characterization. This section reviews representative methods (Section 5.1) and outlines future research directions for intelligent PHM (Section 5.2). Figure 5 summarizes the overall structure of this section.

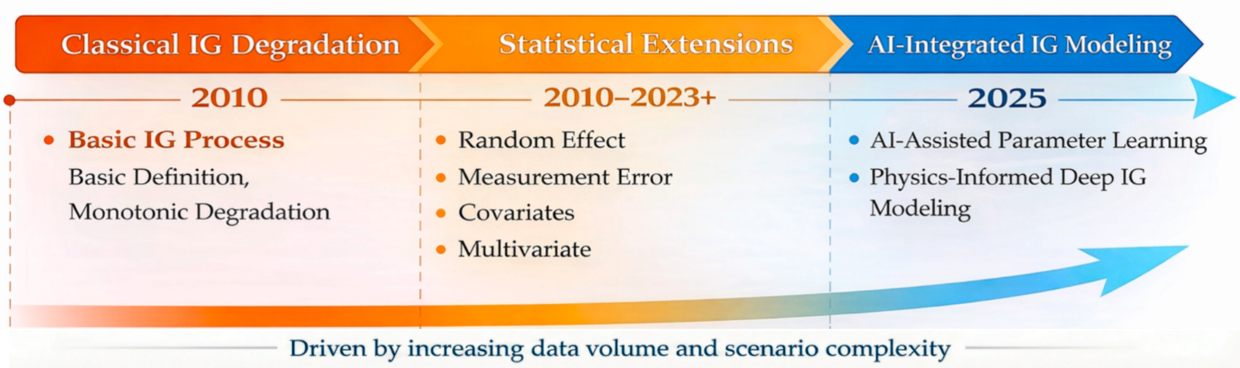


Figure 4: Evolution of IG-process-based degradation modeling.

5.1. Current Research Trends

Figure 6 outlines a general workflow of AI-integrated IG-process-based PHM. Given degradation data from practical systems, the first step is data preprocessing, including outlier diagnosis, missing-data handling, and feature/health-indicator (HI) construction (Ma et al., 2024). For multi-dimensional and heterogeneous data sources, AI-based tools can be employed to improve robustness in these steps. Next, an AI-integrated IG model is built

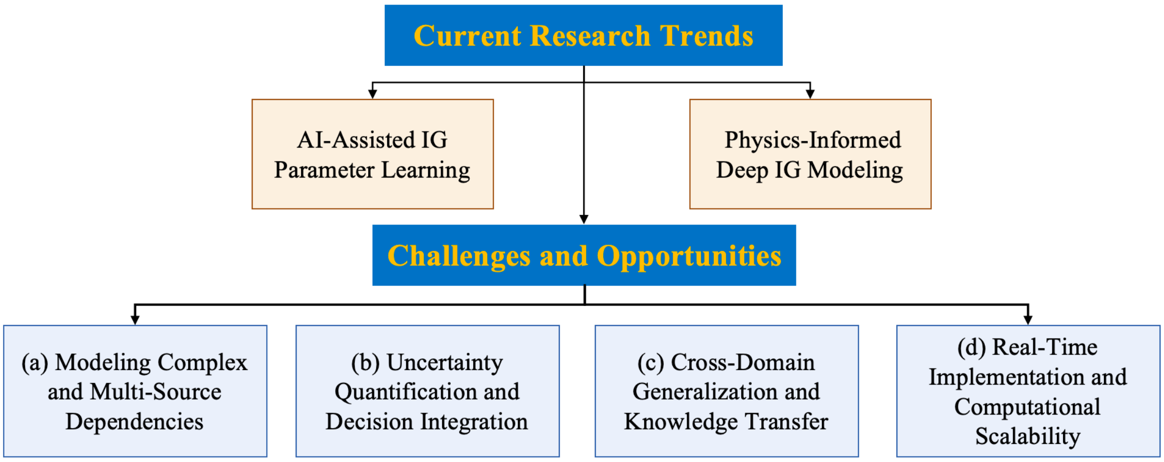


Figure 5: Framework of AI-integrated IG modeling and its challenges in PHM.

for degradation modeling. Recent studies mainly follow two paradigms for integrating AI with the IG process: (1) AI-assisted IG parameter learning and (2) physics-informed deep IG modeling. After the model is fitted using training data, it can be used to generate predictions for test data and support downstream PHM tasks (see Section 4). Next, we review these two paradigms in detail.

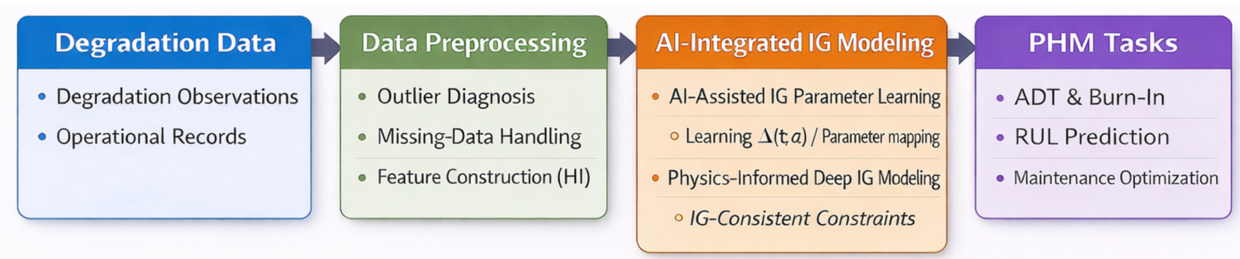


Figure 6: A general workflow of AI-integrated IG-process-based PHM.

(1) AI-Assisted IG Parameter Learning. In conventional IG-based degradation modeling, the shape function $\Lambda(t;\alpha)$ is typically defined in a predefined functional form, such as a power or exponential expression. However, these simple parametric forms have limited capacity to represent complex or nonstationary degradation dynamics observed in real systems. Several studies have defined $\Lambda(t)$ as a nonlinear function learned by neural networks, for example:

$$\Lambda(t) = \sum_{g=1}^{n} \omega_g h_g, \tag{3}$$

where h_g is the output of the gth hidden neuron, and ω_g is its associated weight. Duan et al. (2023) first proposed an artificial neural network-supported IG modeling and reliability estimation approach based on a single-input single-output structure, while Pang et al. (2024) extended this idea to ADT, enabling joint modeling of degradation and lifetime data. Because degradation observations are often limited in size, existing studies typically employ relatively simple neural networks with a small number of parameters to model deterministic components such as the shape function, in order to avoid overfitting and preserve the statistical interpretability of IG parameters. This strategy improves adaptability to nonlinear or multi-stress degradation patterns and has also been extended to other stochastic-process-based degradation models (Liu and Wang, 2021; Hu et al., 2025). However, most of these studies do not systematically address hyperparameter selection and largely rely on empirical settings; moreover, model outputs are usually deterministic, leading to limited uncertainty quantification and potentially unstable estimation under small-sample or complex operating conditions.

(2) Physics-Informed Deep IG Modeling. Unlike approaches that directly use neural networks to approximate IG process parameters, this class of methods explicitly embeds the statistical properties or physical constraints of the IG process within deep learning architectures. Such integration allows the learned degradation trajectories to maintain physical consistency while capturing complex nonlinear behaviors. Zhang and Chen (2025) developed a physics-informed transformer network based on a multi-stage IG degradation model for the RUL prediction of power devices. The model incorporates multi-stage IG representations into the network and enforces monotonicity, increment-distribution, and FPT consistency constraints during training, achieving a joint optimization between feature extraction and physical regularization. The results demonstrated improved stability and interpretability under multi-stage degradation scenarios, as well as better physical realism and statistical calibration in RUL prediction. More broadly, physics-informed IG frameworks often adopt more complex network architectures to leverage rich, high-dimensional sensing data in practical monitoring scenarios. To mitigate hyperparameter sensitivity, existing studies typically use adaptive weighting schemes to balance constraint terms against the primary task loss, thereby reducing reliance on manual weight tuning and stabilizing training and prediction (e.g., via gradient-normalization-based strategies). Meanwhile, they introduce architecture-level designs to reduce redundant computations, improving practical usability without sacrificing predictive performance.

5.2. Challenges and Opportunities

Despite recent progress, current AI–IG models remain limited in scope. Most existing approaches still rely primarily on numerical degradation trajectories, offering limited support for multi-source information integration, uncertainty representation, cross-domain generalization, and real-time scalability in practical PHM systems. The following points highlight the main challenges and opportunities for future research.

(a) Modeling Complex and Multi-Source Dependencies. Modern PHM systems increasingly generate heterogeneous, multi-source health information, including numerical degradation measurements as well as non-numerical data such as images, video streams, and unstructured textual records. While traditional multivariate IG and copula-based models can characterize dependencies among a limited number of numerical PCs, they are not designed to handle heterogeneous data types or to capture time-varying and cross-modal degradation relationships (Yi et al., 2025; Shen et al., 2019b). AI–IG integration offers a promising direction: neural and graph-based models can extract latent representations from diverse data modalities, whereas the IG process provides a probabilistic framework for interpretable degradation dynamics and first-passage behavior. Future research should focus on hierarchical or hybrid IG frameworks that fuse learned neural features—derived from images, video, or textual sources—with interpretable IG-based dependency structures. Such approaches may enable scalable and explainable modeling of cooperative degradation behaviors across multiple variables, sensing modalities, and data types.

(b) Uncertainty Modeling and Decision Integration. Most existing IG-based RUL prediction methods derive probabilistic lifetime estimates through the FPT distribution but rely on point-estimated parameters, ignoring model and data uncertainties (Zhuang et al., 2024; Li et al., 2022; Huynh, 2021). Such simplifications often lead to overconfident RUL predictions, limiting their applicability in risk-sensitive maintenance and logistics planning. To address this issue, it is essential to characterize multi-source uncertainties—arising from parameter estimation, model structure, and input perturbations—into the RUL distribution to support reliable decision-making. Bayesian deep learning and variational inference offer efficient means to approximate the posterior distributions of IG parameters or latent degradation states, enabling uncertainty-aware RUL prediction in complex systems (Pan et al., 2024; Faizanbasha and Rizwan, 2025). Furthermore, reinforcement learning and decision-oriented neural architectures can be embedded into IG reliability frameworks to directly link uncertainty quantification with maintenance optimization, yielding adaptive strategies that minimize long-term cost or failure risk. Future research should focus on hybrid inference–decision frameworks that leverage AI for efficient uncertainty propagation and decision optimization, while using the IG process as a probabilistic foundation for risk-aware maintenance planning. At the system level, aggregating RUL uncertainty across dependent components may enhance mission reliability and spare-parts management, leading to data-driven, risk-sensitive maintenance policies (Nemani et al., 2023).

(c) Cross-Domain Generalization and Knowledge Transfer. In practical PHM applications, degradation data are often scarce, highly heterogeneous, and domain dependent, making it difficult to construct reliable IG models across different operating conditions and systems (Fallahdizcheh and Wang, 2025; Wu et al., 2023). Modeling each dataset separately often causes information loss and unstable parameter estimates, reducing generalization and making it hard to transfer degradation knowledge (Cheng et al., 2023). AI-based methods offer a potential solution through adaptive representation learning, enabling knowledge transfer and sharing across different systems. Neural, meta-learning, and transfer-learning approaches can extract shared latent structures or parameter mappings to capture common degradation mechanisms while maintaining domain-specific adaptability (Lin and Chang, 2021). Future work should focus on interpretable cross-domain parameter mapping—such as relationships between drift and volatility functions across varying loads, environments, or equipment—and on using pre-trained models for knowledge distillation to accelerate adaptation in new domains. In addition, establishing standardized validation protocols for small-sample and cross-domain scenarios will be essential to ensure the robustness and comparability of AI-integrated IG models.

(d) Real-Time Implementation and Computational Scalability. As IG-based models are increasingly applied in practical PHM systems, challenges related to computational efficiency, real-time adaptability, and model traceability have become more prominent. Traditional inference methods for the IG process, such as the ML and EM algorithms, can be computationally expensive for large fleets or online monitoring scenarios, particularly when frequent updates of latent variables or multi-source inputs are required. The integration of AI modules further increases model complexity and data throughput, introducing additional challenges in interpretability, robustness, and auditability for industrial deployment. To address these issues, future research should develop scalable learning and inference strategies that balance efficiency and transparency. Recursive or incremental estimation can enable online parameter updates from streaming data, supporting adaptive health assessment under dynamic conditions (Ren et al., 2024). Approximate and distributed inference methods —such as variational or composite likelihood approaches—can further improve computational scalability in large or high-dimensional systems (Cao et al., 2025; Wang et al., 2025). Integrating interpretable AI tools and visualization-based diagnostics into the AI–IG workflow will enhance model transparency and auditability, while end-to-end frameworks linking data processing, inference, and maintenance decisions are essential for achieving real-time, deployable PHM applications.

Looking ahead, several potential "killer applications" may further drive the development of IG-process-based and AI–IG hybrid PHM in the next 5–10 years. First, real-time health management for large-scale battery energy storage systems requires reliable degradation forecasting and calibrated uncertainty to support risk-sensitive operational decisions under fast-changing conditions. Second, fleet-level PHM for connected assets (e.g., vehicles, industrial robots, and distributed power devices) calls for scalable degradation modeling that can accommodate unit-to-unit heterogeneity and nonstationary usage profiles. Third, safety-critical monitoring in complex infrastructures (e.g., smart manufacturing lines and energy networks) benefits from interpretable stochastic degradation models that can be integrated with AI-based perception modules while maintaining probabilistic consistency (Zhang et al., 2025b).

6. Conclusions

This paper provides a comprehensive review of recent advances in the application of the IG process within PHM. First, the theoretical foundations and modeling aspects of the IG distribution were revisited, followed by an overview of its extended forms, including random effects, measurement error, covariates, and multi-PC modeling. Second, common approaches for parameter estimation, model diagnosis were summarized. Furthermore, the applications of the IG process in ADT design, burn-in test, RUL prediction, and maintenance decision-making were systematically reviewed. In addition, the paper discussed emerging research on AI-integrated IG modeling, highlighting recent integration paradigms, methodological challenges, and future directions for intelligent PHM. Overall, the IG process provides a unified and interpretable probabilistic framework for degradation modeling, and its integration with AI is expected to further improve flexibility, scalability, and decision support in intelligent PHM systems.

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Competing Interests

The authors declare that they have no competing interests.

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